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ABSTRACT

Through a review of the literature, this paper explores the viability of the rotation of canonical correlation analysis results. The similarities and dissimilarities between factor analysis and canonical correlation analysis are examined. The logic supporting a preference for the rotation of structure coefficients as opposed to function coefficients is presented, along with some examples. The primary advantage of rotation appears to be an improvement in the interpretability of the results. A second advantage of rotation is that it can be used as an invariance procedure to reveal the common structure between samples. However, these are two distinct applications. Canonical analysis focuses on extracting orthogonal solutions that maximize the relationships between the two sets of variables. A small hypothetical data set is used to illustrate the results of different types of rotation using the ORSIM2 computer program. Concrete examples are used to show the following: (1) the sum of the canonical correlations after rotation is the same as before rotation; (2) the sum of squared multiple correlations after rotation is identical to the sum of pre-rotation squared canonical correlations; and (3) the total predictive power of each variable is unchanged by rotation. Six data tables are included. (Author/TJH)

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THE EFFECTS OF ROTATION IN CANONICAL CORRELATION ANALYSIS

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ABSTRACT

This paper explores the viability of the rotation of canonical correlation analytic results. The similarities and dissimilarities between factor analysis and canonical correlation analysis are examined. The logic supporting a preference for the rotation of structure coefficients as opposed to function coefficients is presented, along with some examples. The primary advantage of rotation appears to be an improvement in the interpretability of the results. A second advantage of rotation is that it can be used as an invariance procedure to reveal the common structure between samples. However, these are two distinct applications. A small hypothetical data set is used to illustrate the results of different types of rotation using the ORSIM2 computer program. Concrete examples are used to show the following: (a) the sum of the canonical correlations after rotation is the same as before rotation, (b) the sum of the squared multiple correlations after rotation is identical to the sum of the pre-rotation squared canonical correlations, and (c) the total predictive power of each variable is unchanged by rotation.

THE EFFECTS OF ROTATION IN CANONICAL CORRELATION ANALYSIS

Although canonical variate analysis has been available to researchers since its development by Hotelling in 1935, it remains a somewhat infrequently used statistical methodology. The original use as proposed by Hotelling was to find the weighted average of several criterion variables forming the most predictable criterion for a test battery. Because researchers had little interest in determining a criterion just because it could be predicted, most behavioral scientists have rejected this original use of canonical variate analysis (Darlington, Weinberg, & Walberg, 1973).

Major contributions to canonical analysis were subsequently made by Wilks in 1935 and Bartlett in 1941, as regards effect size estimation and significance testing, respectively. However, the mathematical complexity of canonical analysis and the required calculations discouraged all but the most sophisticated researchers from applying the method (Wood & Erskine, 1976). Thompson (1984) stated a second explanation for the lack of use of canonical analysis. He noted that the technique is rarely used because of the difficulties the researcher encounters in trying to interpret canonical results.

More recently, however, with the advent of high-speed computers and the inclusion of canonical analysis in major statistical packages (Krus, Reynolds, & Krus, 1976), canonical analytic results are being reported more frequently in research journals. Wood et al. (1976) report over 30 applications of this method in their review of the literature between 1965 and 1976.

There is also evidence that the need for canonical correlation analysis is becoming more recognized and thus its frequency as the statistical method of choice is increasing. Thompson (1988) argues that the use of multiple univariate tests in situations where multivariate methods are more appropriate inflates the experimentwise Type I error probability and distorts the reality and generalizability of results. Moreover, statistical significance, although not of paramount concern, may be obtained in the multivariate case of canonical correlation when it is not obtained in the use of multiple univariate tests (Fish, 1988).

The Relationship of Canonical Analysis to Other Parametric Methods

It is generally recognized (Darlington et al. , 1973; Knapp, 1978; Levine, 1977; Thompson, 1984, 1988) that canonical correlational analysis is the most general case of the linear model. Darlington et al. (1973) state that canonical variate analysis is the most general technique in the array of parametric techniques and encompasses ANOVA, MANOVA, and multiple regresssion as special cases. Through the use of dummy variables on either the dependent or independent side, canonical variate analysis can be employed to perform multiple discriminant analyses or contingency table analyses. Thompson (1988) goes even further in illustrating how canonical analysis can be used to produce results from both univariate and multivariate analyses. Furthermore, Thompson states emphatically that canonical analysis is a preferable technique because it does not demand the

discarding of variance that many researchers engage in when converting continously scaled variables to nominally scaled variables.

In considering the question of whether the results in canonical analysis may be rotated, the relationship between canonical analysis and factor analysis must be explored. On one side of the argument, both techniques share a common foundation in the general linear model. The mathematics for function extraction are the same, although different matrices are used in the calculations (Thompson, 1984). On the other side of the argument lies the fact that canonical analysis deals with two sets of variables and factor analysis deals with only one set of variables. In canonical analysis, there are two sets of component loadings which reflect both the relations between the sets and relations within the sets (Thorndike, 1976). Thus, whereas the factor model establishes linear functions that have maximum variances with respect to reproducing a matrix of associations, the canonical model selects linear functions that have maximum covariances across variable sets (Cooley & Lohnes, 1971).

Canonical correlation analysis permits the researcher to examine the relationship between two sets of variables by deriving linear combinations of variables from each of the two sets so that the correlation between the two sets is maximized. This procedure is much like that of principal components analysis, in that the first factor accounts for the maximum amount of variance in the set of variables, and each subsequent factor extracted accounts for as much of the variance as possible that has not been accounted for by the preceding

factors. Similarly, in canonical analysis each subsequent function extracted accounts for the maximum amount of correlation between the two sets of variables.

These similarities and dissimilarities play a major role in establishing the correctness of the role of rotation in canonical analysis. Although Levine (1977) suggested that there were few remaining puzzles to be solved in canonical analysis, he went on to suggest that "in the case of the factor comparison techniques, there are large numbers of solutions to essentially different puzzles and the data analyst must be alerted to the importance of the selection of the appropriate puzzle solution" (p. 8). Among these puzzles and extensions of canonical analysis was rotation.

The Logic Underlying the Rotation of Structure Coefficients

Canonical correlation analysis simplifies the total correlation matrix involving two sets of variables by reducing the two sets of variables to a smaller quadruple-product matrix (Thompson, 1984). The function coefficients (akin to beta weights or factor pattern coefficients) are extracted from this quadruple-product matrix.

Bentler and Huba (1982) state that two criteria must be met if the results of a canonical correlation analysis are to be useful: "It must be ascertained not only how well a small number of canonical variates can summarize statistical association through the canonical correlations, but also how well the variates can be conceptualized and interpreted" (p. 21). According to these two researchers, the variates

are often statistically adequate but theoretically meaningless. Thus, experimentation with rotation has been brought about by the need to provide a better theoretical summary of the results.

A basic feature of canonical correlation analysis is that there is no correlation within and between sets of unrotated canonical variates except for the single pairwise linkage of one variate from one set to the other variate for the other set on a given function. After rotation, however, the variates in each set will be correlated to the variates in the other set. In other words, the off-diagonal elements in the canonical correlation matrix will no longer be exactly zero. Since relationships are often complex, it may well be that this new canonical correlation matrix more realistically reflects these relationships than does the original canonical correlation matrix in which the off-diagonal elements are zero.

Once the decision to rotate the results is made, the question which must be answered is whether to rotate the function coefficients or the structure coefficients. The function coefficients are the "weights" which are used to multiply times each person's data to create the new canonical variates. The structure coefficients are analagous to factor structure coefficients in principal components analysis. A structure coefficient is the correlation between the latent predictor or criterion variate scores and the originally observed variables used to create the composite scores. As such, structure coefficients indicate the contribution a single variable makes to the explanatory power of the set of variables.

As proved by Cliff and Krus (1976), the pre-rotation predictive power and the post-rotation predictive contributions of each variable are equal under a Varimax rotation. Thompson uses this fact to provide a convincing logic for the rotation of the structure coefficients (1984). In the case of structure coefficients, the sum of these squared coefficients are communality coefficients, which have psychometric meaning, whereas no such claim can be made for the function coefficients. The communality coefficient indicates what proportion of each variable's variance is reproducible from the total canonical results. A squared canonical structure coefficient indicates the proportion of the variance linearly shared by a variable with the full canonical solution.

Levine (1977) is equally emphatic in his claim that one must interpret the structure coefficients rather than the function coefficients, although his argument makes no specific reference to rotation. The multicollinearity or intercorrelation among the variables within each set can result in the suppression of function coefficients of one variable by another variable. Once the first variable has made its contribution to the composite, there may be little or no contribution that a second correlated variable can make. Thompson (1988) presents (Table 1) the following example of different conclusions that result from the interpretation of function coefficients as compared to structure coefficients.

INSERT TABLE 1 ABOUT HERE.

As can be seen from the data in Table 1, the function coefficients indicate that all four variables contribute more or less equally to the relationship between the two sets of synthetic variable scores. However, an interpretation of the structure coefficients shows that variables ZX and ZB make almost no contribution to the variance of the canonical composites.

Darlington et al. (1973) discuss a different logic regarding the interpretation of function and structure coefficients. Based on the fact that the function coefficients are weights and the structure coefficients are correlations, they indicate that sampling error alone would dictate the interpretation of structure coefficients because it can be inferred that the standard errors of weights are frequently much higher than those of correlations. Consequently, when the variables in a set are highly intercorrelated, the differences between the weights and correlations will be at a maximum. In such circumstances, the function coefficients must be treated with suspicion and the correlations or structure coefficients should be interpreted. However, Thompson's (1989) empirical results in a Monte Carlo study suggest that this feature of structure coefficients should not be exaggerated.

Further support for the interpretation of structure coefficients comes from Reynolds and Jackosfsky (1981). Their research example illustrates a situation in which the interpretability of the results by rotating the function coefficients is not enhanced, but the interpretability of the results by rotating the structure coefficients is

greatly improved. Based on the fact that structure coefficients are standardized and constrained to be less than one, the researchers conclude that the rotation of structure coefficients will result in less extreme solutions.

Bentler et al. (1982) also state a preference for rotation of the loadings (structure coefficients). According to them, "the goal of rotation in canonical correlation is to introduce as many zero or near-zero loadings as possible, while having the cross-variate correlations remain interpretable" (p. 28). In such a rotation, there are no constraints on the correlations between the two sets of variables.

A Brief History of the Development of Rotational Procedures

Cliff et al. (1976) were among the pioneers in the rotation of canonical solutions. Their rationale in suggesting that rotation would produce a more desirable solution which could interpreted more meaningfully lay in the fact that canonical correlation analysis, like factor analysis, involves solving the characteristic equation for latent roots and vectors and therefore would have its optimizing properties preserved in rotation, just as takes place in factor analysis.

Krus et al. (1976) provide an example of the substantive interpretation of both the unrotated results and the rotated results from a Varimax rotation of the function coefficients. Their conclusion was that the interpretations of both were partially in agreement, but the rotated solution was easier to interpret because it simplified the

overall configurational pattern. Additionally, the rotated results were more self-consistent than the unrotated results. In summary, it appeared that rotation enhanced the isolation of the relationships and thus made them easier to interpret.

In one of the earliest references to rotation, Darlington et al. (1973) provided an original assertion that canonical variates could be interpreted as factors and the variates in either set could be rotated like any other factors without regard to the other set. This represents an important recognition of the fact that the researcher should or could consider the sets of variables independently.

Thorndike (1976) also approached the question of rotation from the perspective of how to handle the relationships between the two sets of variables. He set forth three possible approaches to the rotation of canonical components: (a) rotation of the entire set of components, (b) rotation of each set of components independently to new locations, and (c) the selection of one set for rotation and the subsequent rotation of the second set such that there was a maximum correlation with the rotated components of the first set. For the first approach, the conclusion reached was that it was a logical contradiction to try to obtain a combined simple structure for the total set of components, given that the two sets of variables had been kept separate for a reason. The primary flaw in the second approach was that the major criterion of canonical analysis, that of extracting orthogonal functions that maximize the correlations between the sets of variables, was sacrificed for the sake of clarity of structure within

each set. The third approach was deemed most advantageous due to the fact that, although the optimization of relationships between the sets would not hold precisely, this criterion would be better met than in the other two alternatives.

The most substantial work regarding the problem of how to address the relative importance of the two sets of variables has been conducted by Bentler et al. (1982). They indicate that the two sets of variables can be rotated simultaneously if the researcher has no particular reason to be concerned with simplification in one set versus simplification in the other set. In such a rotation, the influence of each set of variables on the rotated solution is proportional to the number of variables in the set relative to the total number of variables. For example, if there are five predictor variables and 15 criterion variables, the predictor variables will have 5/20 = 1/4 relative influence on the solution and the criterion variables will have 15/20 = 3/4 relative influence on the solution. In the situations where one set of variables is already understood to have simple structure and the other set does not, the researcher might wish to override this approach and let a designated set of variables determine the rotation.

Huba, Palisoc, and Bentler (1982) developed the FORTRAN program ORSIM2 which provides different rotations that honor the membership in the variable sets by setting the parameter α ($0 \leq \alpha \leq 1$) that represents the relative influence that the variables in matrix one (usually the predictors) will have on the solution: (a) rotation of the two sets of coefficients by weighting each set proportionally to its

number of rows relative to the sum of the number of rows in the total set (as in the preceding example, α = 1/4), (b) rotation of the two sets by maximizing the criterion variables only (α = 0), (c) rotation of the two sets by maximizing the predictor variables only (α = 1), and (d) rotation of each set independently for simple structure. The form of the rotation chosen should be consistent with the theories prevalent in the conceptual literature. For example, if one set of variables is known to have simple structure already, the researcher's interest might lie in weighting the other set of variables to achieve simple structure in that set.

The first three of these rotations are referred to as symmetric rotations, whereas the fourth rotation is labeled an asymmetric rotation. Bentler et al. (1982) state a preference for the asymmetric rotation because "the loading matrices tend to be simpler in form with an asymmetrical rotation, and consequently it is easier to interpret the variables in terms of the canonical variates and vice versa" (p. 33). In all instances, the canonical correlations are adjusted after the rotation and are referred to as quasi-canonicals.

Additionally, a maximum likelihood interbattery factor analysis is calculated by the computer program the researchers developed. The factor loadings used in the interbattery factor analysis are the supermatrix of unrotated canonical variate loadings formed by placing the two matrices of canonical loadings for the two variable sets end to end and then rescaling by the square root of the canonical correlations. The advantage that the interbattery factor analysis

offers is that it will reveal the common latent variable sources of variance for a set of variables that is grouped into two sets (Huba, Newcomb, & Bentler, 1981).

Bentler et al. (1982) advocate that several different rotations should be performed to ascertain whether they support the same theoretical conclusions. Generally speaking, they suggest rotating only those functions which are statistically significant. However, Guertin, Guertin, and Ware (1981) gave evidence that the stability of factor loadings was significantly affected by the number of factors that were rotated and concluded that the rotation of too few factors may lead to serious distortions in the results.

In a review of the literature regarding rotation to simple structure to achieve better interpretability, Thompson (1984, 1988) takes a rather dim view of rotation to the Varimax or similar criteria which treat the variables as one set, rather than two distinct sets. He suggests instead that one employ a rotation procedure, such as the procedure in the ORSIM2 program (Huba et al., 1982), which honors membership in variable sets. In either case, the maximal correlational property of the canonical analysis is lost because the matrix of canonical correlations between the two sets is no longer diagonal. However, Thompson concludes that "these difficulties may represent reasonable tradeoffs in favor of improved interpretability and possibily more invariant results" (p. 40).

Rotation as an Invariance Procedure

Not only does rotation appear to improve the interpretability of the canonical solution, but a second advantage of rotation is that it can be used as confirmatory rotation to reveal the structure common to two samples. Wilkinson (1977) provides an example in which the original solution, which is not easily interpretable in terms of existing theory, is rotated to interpretable form (exploratory rotation) and then the replication solution is fitted to this rotated structure (confirmatory rotation).

Like all correlational methods, canonical analysis tends to capitalize on chance. Thompson (1981) notes that most extraction procedures reach their criterion by such capitalization. In the case of factor analysis, Gorsuch (1983) recommends that an estimate of the magnitude of this capitalization can be found by randomly splitting the sample in half and comparing the factors extracted from both data sets. Thompson (1981) asserts that this same logic can be generalized to all correlational methods, all of which capitalize on chance.

Thompson (1986) provides a discussion of an empirical method developed by Kaiser, Hunka, and Bianchini for "relating" factors based upon data from different samples. The method consists of projecting the two sets of factors into the same space and calculating the cosines of the angles among the factors across the two solutions. These cosines are, in fact, correlation coefficients and will provide a measure of the relatedness of the two sets of factors. Thompson (1981, 1986)

states that these coefficients are analagous to test-retest coefficients and has called them invariance coefficients, although they may also be utilized for substantive interpretations and as adequacy coefficients.

Using the RELATE program by Veldman, the function coefficients from one sample are input as matrix A and the function coefficients from the other sample are input as matrix B. Matrix B is then rotated to a position of "best fit" with matrix A. Thompson (1982) states that although

> the rotated canonical function weights may no longer optimize the canonical correlation, . . . the "best fit" rotation presented as an invariance procedure does not violate the basic logic of canonical analysis, since the researcher will not be interested in interpreting the rotated canonical functions. (p. 5)

When the primary concern is to investigate the similarity of function equations, the function coefficients should be rotated, whereas when the major concern is to examine the similarity of the distribution of variable variances across equations, the structure coefficients should be rotated (Thompson, 1984).

It should be observed that little is known about the characteristics of the various invariance estimates. As a result, Thompson (1984) suggested that the researcher might wish to employ several strategies to obtain both "lower bound" and "upper bound" estimates of the degree of capitalization on sampling specificity. Thompson (1986) used Monte Carlo techniques to generate a partial

test distribution for factor cosines, since the minimally acceptable values for these cosines have not been clearly established. The reader is reminded that the cosine for 0 degrees is 1 and the cosine for 90 degrees is 0. Therefore, the greater the relationship between the factors, the closer to 1 the cosine will be.

Huba, Wingard, and Bentler (1979) provide an example of the cross-validation method of invariance in that they conduct the same analysis on two different samples, rotate each to "simple structure," and then evaluate the two sets of results for convergence by the use of congruence coefficients.

Comparisons Between Unrotated and Rotated Coefficients in a Hypothetical Research Situation

In this hypothetical research situation, there are two criterion variables, Job Satisfaction and Self-Actualization, and two predictor variables, Autonomy(measured as the difference between the need for autonomy and the fulfillment of this need), and Participation (measured as the difference between the need for participation and the fulfillment of this need). In the case of the predictor variables, larger scores reflect frustration; in the case of the criterion variables, larger scores indicate a positive response. This leads to inverse relationships between the criterion and the predictor variables.

First, a comparison between the function coefficients and the structure coefficients will illustrate the importance of interpreting and rotating the structure coefficients. As can be seen from the unrotated

results presented in Table 2, the function coefficients and the structure coefficients are reversed on functions I and II. The unrotated function coefficients indicate that little contribution to function I is made by Job Satisfaction and that on function II Job Satisfaction is relatively equal in contribution to Autonomy and Participation. In contrast, the unrotated structure coefficients show that Job Satisfaction is a moderate contributor to function I ($r = .54$) and the major contributor to function II ($r = .84$).

INSERT TABLE 2 ABOUT HERE.

With the rotated coefficients in Table 3, there is an improvement in interpretability for both the function and the structure coefficients. Function I can now be identified as primarily Actualization and Participation and function II can be identified as primarily Job Satisfaction and Autonomy. The major difference between the interpretation of the function and structure coefficients lies in alternate decisions across coefficient type regarding what is the third versus the fourth largest contributor to the solution.

INSERT TABLE 3 ABOUT HERE.

Table 4 indicates that all six rotation strategies achieve the same theoretical results in terms of the structure of the functions. Before rotation, function I is a composite of Actualization ($r = 1.0$), Autonomy

(r = -.86) and Participation (r = -.95), whereas function II consists primarily of Job Satisfaction (r = .84). However, after each of the six different rotations, the loadings tend to cluster high on function I for Actualization and Participation and high on function II for Job Satisfaction and Autonomy. The greatest difference between the high and low loadings occurs for the asymmetric rotation, reflecting the fact that it is a whole different class of rotation (asymmetric as against symmetric).

INSERT TABLE 4 ABOUT HERE.

In their early work on rotation of coefficients using a Varimax rotation, Cliff et al. (1976) showed two important mathematical relationships: (a) the sum of the rotated canonical correlations is equal to the sum of the original canonical correlations, and (b) the total predictable variance in each set of variables remains the same after rotation.

Bentler et al. (1982) also make these same points when they state that "symmetry of prediction is maintained when investigating squared multiple correlations with symmetrically rotated variates." The results in Table 5 generated by the ORSIM2 rotations of the structure coefficients can be examined to verify that these properties hold true under all rotations except the asymmetric one.

INSERT TABLE 5 ABOUT HERE.

As the data in Table 5 indicate, the sum of the canonical correlations after rotation is the same as before rotation except in the asymmetric rotation. The sum of the canonical correlations (on the diagonal) before rotation is 1.4836, whereas the sums after rotation range from 1.4836 to 1.4837, within rounding error of the sum before rotation. Although the sums are the same, the rotated canonical correlations are spread more evenly among the new pairs of variates, in contrast to the unrotated solution in which the maximization property of canonical analysis generates a much higher canonical correlation for the first function. In the case of the asymmetric rotation, the sum of 1.3650 indicates that some portion of the maximization property of canonical analysis has been lost, as expected.

The sums of the squared canonical correlations plus the squares of the off-diagonal elements after rotation (1.1597 to 1.1599) are, in all cases, equal to the sum of the pre-rotation squared canonical correlations (1.1597). These sums are the same as squared multiple correlations. Since rotation brings about new correlations among the variates that did not exist before rotation (off the diagonal), the squares of these off-diagonal elements must be included in the calculation of the total predictive variance or total squared multiple correlations. Thus, the total predictable variance in each battery remains the same after rotation, whether one sums across the columns or down the rows. In other words, the sums of the squared multiple correlation values for predicting a given domain from the other are

equal across rotation methods.

Finally, the contribution of each variable to the canonical solution is examined by looking at the sum of the squared structure coefficients for each variable, i. e., the communality coefficients. The results in Table 6 show that the predictive power of each variable before rotation is equal to its predictive power after rotation since both sum to 1.000, within rounding error.

INSERT TABLE 6 ABOUT HERE.

Summary

Canonical analysis focuses on extracting orthogonal solutions which maximize the relationships between the two sets of variables. This focus is abandoned to some extent when rotation of the coefficients is performed. Although rotation to simple structure does produce results that can be interpreted more easily, the researcher must weigh this loss of maximization against the gain in interpretability. The evidence favors rotating structure coefficients rather than function coefficients, but the researcher may benefit from a rotation of both sets of coefficients. More work is needed to explore the relative benefits of rotation of canonical results in various applied real research situations.

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Table 1

Function I

	Function Coeff.	Structure Coeff.
ZX	-1.44986	-.69607
ZY	1.04101	-.00884
ZA	-1.58021	-.61831
ZB	1.24215	.08146

Table 2

Unrotated Results

Variables	Function Coefficients		Structure Coefficients	
	Func. I	Func. II	Func. I	Func.II
Job Satisfaction	.0031	1.1826	.5361	.8441
Actualization	.9983	-.6340	1.0000	- .0026
Autonomy	-.4275	-1.2567	-.8652	-.5013
Participation	-.6656	1.1485	-.9467	.3220

Table 3

Varimax Rotated Results

	Function Coefficients		Structure Coefficients	
	Func. I	Func. II	Func. I	Func. II
Job Satisfaction	-.5067	1.0684	.1200	.9928
Actualization	1.1739	-.1416	.9035	.4287
Autonomy	.1557	-1.3179	.5647	-.8255
Participation	-1.0954	.7492	-.9931	-.1175

Table 4

Rotated Structure Coefficients

Type of Rotation	Variables	Function I	Function II
Unrotated	JSAT	.5361	.8441
	ACT	1.000	.0026
	AUT	-.8652	-.5013
	PART	-.9467	.3220
Varimax	JSAT	.1200	.9928
	ACT	.9035	.4287
	AUT	-.5647	-.8253
	PART	-.9931	-.1175
Proportionally	JSAT	-.0803	.9967
Weighted	ACT	-.8857	.4643
α = .5	AUT	.5313	-.8471
	PART	.9876	-.1570
Criterions	JSAT	-.2779	.9606
Weighted	ACT	-.9606	.2779
α =1.0	AUT	.6899	-.7238
	PART	.9990	.0436
Predictors	JSAT	.1206	.9927
Weighted	ACT	-.7750	.6320
α = 0.0	AUT	.3512	-.9363
	PART	.9363	-.3513
Asymmetric	JSAT	-.2779	.9606
	ACT	-.9606	.2779
	AUT	.3512	-.9363
	PART	.9363	-.3513
Maximum	JSAT	-.1274	.8077
Likelihood	ACT	-.8301	.4740
Factors	AUT	.5291	-.7398
	PART	.9059	-.2395

Note: The criterion variables were entered first as matrix one. Consequently, the alpha values refer to the weight for the criterion variables.

Table 5

Adjusted (Quasi-) Canonical Correlations (on Diagonal)

after Rotation of Structure Coefficients

			Squared		
Rotation Type	Func. I	Func. II	Func. I	Func. II	Sum
Unrotated	.9138	0.000	.8350	0.000	.8350
	0.000	.5698	.0000	.3247	.3247
Sums	*Diagonal = 1.4836*		.8350	.3247	**1.1597**
Proportional	.8389	-.1420	.7038	.0202	.7240
	-.1420	.6447	.0202	.4156	.4358
Sums	*Diagonal = 1.4837*		.7240	.4358	**1.1598**
Criterion Only	.8868	-.0926	.7864	.0086	.7950
	-.0926	.5969	.0086	.3563	.3649
Sums	*Diagonal = 1.4837*		.7950	.3649	**1.1599**
Predictors Only	.7755	-.1687	.6014	.0285	.6299
	-.1687	.7081	.0285	.5014	.5299
Sums	*Diagonal = 1.4836*		.6299	.5299	**1.1598**
Asymmetric	.7796	-.4326	.6078	.1871	.7949
	.1486	.5854	.0221	.3427	.3648
Sums	*Diagonal = 1.3650*		.6299	.5298	**1.1597**

Table 6

Communality Coefficients

(Squared Structure Coefficients)

Variable	Before Rotation			After Rotation (Varimax)		
	Func.I	Func.II	Sum	Func.I	Func. II	Sum
Job Satisfaction	.287	.713	1.000	.0144	.9857	1.0001
Actualization	1.000	.000	1.000	.8163	.1838	1.0001
Autonomy	.749	.251	1.000	.3189	.6811	1.0000
Participation	.896	.104	1.000	.9862	.0138	1.0000